Letter by PT Adamson* to the Editor with regard to the paper

# A DEPTH-DURATION-FREQUENCY DIAGRAM FOR POINT RAINFALL IN SWA-NAMIBIA

by WV Pitman published in *Water SA* 6(4) 157–162, October 1980.

The author is to be congratulated on providing a procedure for the estimation of storm risk in South-West Africa/Namibia. This correspondent is, however, somewhat concerned at the choice of the log-Gumbel model of event probabilities for there are some implications to which it might be useful to draw attention *"coram populo"*.

Given efficient estimation procedures and reasonably valid assumptions there exist no theoretical reasons to favour any particular probabalistic model of hydrometeorological extremes above another and the choice must inevitably be made on empirical grounds. Classical tests of fit are simply not powerful enough to show reasonable sensitivity to both thick and thin tails whilst the $\chi^2$ test in particular would require additional groupings at the upper extreme of the range to convey a measure of the suitability of the model for extreme value analysis. Tests which can be weighted to show specific sensitivity within a certain range of the data are much more enlightening but even so could not alone be considered to be final arbiters between competing models. Consequently the author's choice of the log-Gumbel model based upon a $\chi^2$ does not take into consideration the properties of the tail of this model which may be irrelevant when the mean is to be estimated but can lead to substantially different decisions in extreme value theory.

In terms of asymptotic extreme value theory a Type II distribution can be transformed to a Type I by the simple transformation: $Z = log\,(x - \xi)$ where $\xi$ is the location parameter of the Type II model. (Johnson & Kotz, 1970.) $Z$ is then commonly said to be distributed as log-Gumbel, although it is to be noted that in the hydrological literature $\xi$ is rarely estimated and is assumed to be zero. It is at once apparent that the theoretical validity of the transform depends upon $x$ being strongly distributed as Type II. This is evidently not the case for Namibian one day annual rainfall maxima. It is recalled that for the generalized form of the three asymptotic extreme value distributions (Jenkinson, 1955) that the shape parameter $K > 0, = 0$ *and* $< 0$ identifies the Types III, I and II respectively. For a random sample of 23 records of one day annual maxima drawn from the same data as used by the author and distributed throughout the territory $K$ values derived by maximum likelihood ranged between 0,67 and −0,44 which suggests that if an asymptotic model were to be applied generally in real space then it should be the Type I (Gumbel). Using a test devised by Otten and Van Montfort (1978) to distinguish between the three asymptotic extreme value distributions it was found that of the 23 records 11 could be said to be distributed as Type II, the rest showing evidence of a limit to the upper tail and consequently best fitted by the Type III asymptote.

Given that the theoretical justification for transform to log space is weak the author then intrinsically proceeds, in my opinion, to an error of assumption. The Gumbel distribution (Type I) assumes a linear relationship between the frequency factor $K(T)$ and the $ln\ ln\ [T/(T - 1)]$ function of recurrence interval. However, this linear relationship does not hold in log space as the author has assumed by using tabulated values of $K(T)$ or $y(T) = -ln\ -ln(1 - 1/T)$ the reduced Gumbel variate. From NERC (1975) it is seen that the event $q$ for a recurrence interval $T$, with $q$ estimated in real space is given by:-

$$q(T) = \mu + \alpha\, y(T) \qquad (1)$$

or

$$q(T) = \bar{x} + \sigma K(T) \qquad (2)$$

where

$$K(T) = -0{,}45 + 0{,}78\,(y(T)) \qquad (3)$$

and $\mu$, $\alpha$, $\bar{x}$ and $\sigma$ are sample estimates of location, scale, mean and standard deviation parameters respectively. If $q(T)$ is estimated in log space then any error in $K(T)$ will be exponentiated generally resulting in considerable over estimation of an event associated with a given risk. In order to illustrate the magnitude of the error $\mu$ and $\alpha$ were estimated from the logarithms of each of the 23 records of one day annual maxima referred to above. $y(T)$ is then calculated as:-

$$y(T) = (ln\, x_i - \mu)/\alpha \qquad (4)$$

and $T$ from the Weibull plotting position as –

$$T = (N + 1)/rank \qquad (5)$$

where $N$ is the sample size in years and rank refers to the position of the event $ln\ x_i$ in the series arranged from largest to smallest. The $K(T)$ values were averaged in intervals of 0,10 probability of exceedence and plotted against $ln\ ln\ (T/(T - 1))$ as shown in Figure 1, which clearly shows the relationship to be non-linear in log space. Boughton (1980) produced an identical plot to Figure 1 for 78 samples of Australian annual flood peak maxima and showed that if $K(T)$ is not corrected in log space the degree of over-estimation of $q(T)$ can be quite dramatic. For example he shows that the mean log-Gumbel estimate of the 100 year event is 3 times those of the log-Normal and a model accommodating the non-linearity shown in Figure 1.

For one day annual rainfall maxima over SWA the implications of Figure 1 are shown in Figures 2 and 3. Using the mean of the 23 log-Normal estimates of $q(T)$ as a standard the mean estimate of the 100 year event provided by the log-Gumbel model is twice as high. The mean estimates of $q(T)$ from a mixture of two log-Normal distributions (Singh *et al* 1972), the Gumbel model and the truncated log-Normal model applied to the partial duration series are also shown. Figure 2 further shows that the Gumbel estimate above a 50 year return interval is relatively low, the reasons for which are obvious from Figure 1. Corrected lestimates of $K(T)$ applicable to the log-Gumbel model applicable to 1 day annual rainfall maxima in SWA are given in Table 1.

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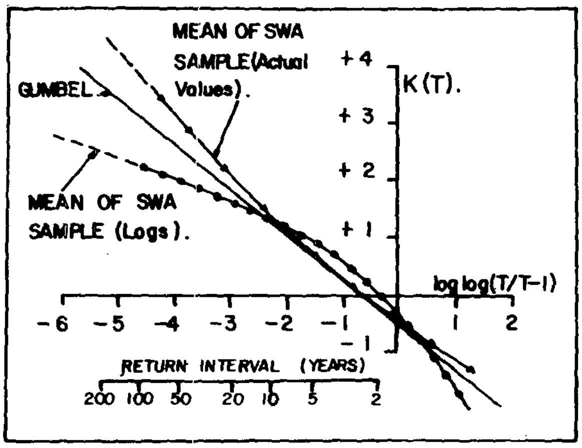


*Figure 1*

| Return Interval (years) | $K(T)$ Tabulated | $K(T)$ Estimated from 23 stations, and corrected for non-linearity |
|---|---|---|
| 2 | −0,17 | 0,08 |
| 5 | 0,72 | 0,85 |
| 10 | 1,31 | 1,30 |
| 25 | 2,04 | 1,60 |
| 50 | 2,59 | 2,00 |
| 100 | 3,14 | 2,27 |
| 200 | 3,68 | 2,50 |

TABLE 1
COMPARISON OF *K(T)* VALUES AFTER CORRECTION FOR NON-LINEARITY IN LOG-SPACE

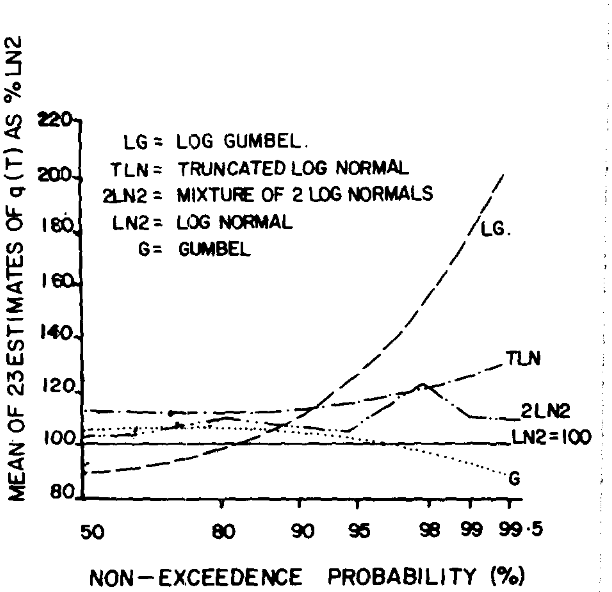


*Figure 2*

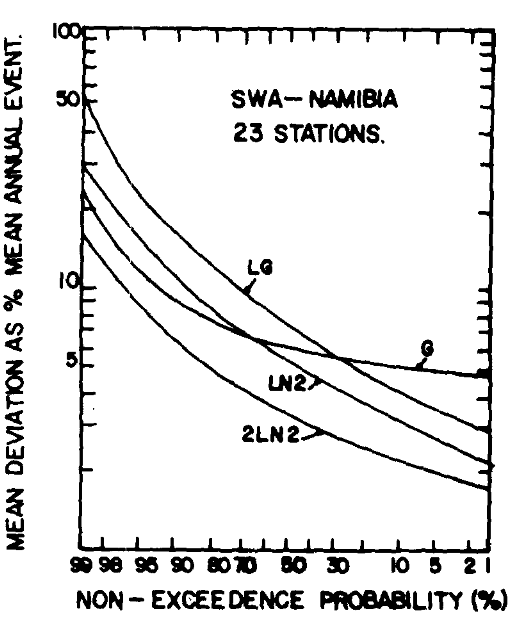


*Figure 3*

Figure 3 further shows that in fact the log-Gumbel model provides the worst fit on the average to the annual series. Deviations from plotted points are expressed in the form:-

$$D = \frac{100}{K} \sum_{i=1}^{K} \frac{F_c - F_o}{F_m} \quad \text{.......................} \quad (6)$$

following Prasad (1970) where $F_c$ and $F_o$ denote the computed and observed events for the same probability P; $F_m$ represents the mean annual event and K = 23. The exercise was repeated for various empirical plotting positions but the overall conclusions remain the same. The mixture of two log-Normal distributions provides the best fit to the sample because this model was fitted by a constrained optimisation procedure to minimise deviations from the empirical Weibull plotting position.

The fact that the author's estimates of *q(T)* are too high is clearly indicated by the fact that only 2,6% of all stations from his sample of over 500 showed maximum daily rainfalls in excess of the computed 100 year value. This does not accord with any theory relating to the arrival of extremes. For example assuming a Poisson arrival process Hall and Howell (1963) have shown that the probability of an event of return interval *T* occuring within a time interval $\Delta t$ may be expressed as:-

$$P = 1 - e^{-\Delta t/T} \quad \text{...................................} \quad (7)$$

Of the data available to the author with records longer than 20 years, below which extreme value analysis could best be described as speculative, the average length is 33 years. Consequently from equation (7) with $\Delta t = 33$ and $T = 100$ it is to be expected that the 100 year event would be equalled or exceeded at 28% of these stations. Of course as a consequence of a lack of total independence between stations this estimate is not statistically clinical but even these effects would not reduce the value to that achieved by the author.

In the light of the foregoing the author's conclusion that extreme rainfalls in SWA-Namibia could be much higher than indicated by earlier work is perhaps questionable. His results remain, however, a considerable contribution to the assessment of storm risk over the semi-arid regions of Southern Africa but must be considered to provide very high upper envelope curves with the implications these carry for design criteria.

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### WV Pitman replies as follows:

The author thanks Mr. Adamson for displaying such a keen interest in the paper. Mr. Adamson's main criticism is aimed at the author's choice of the log-Gumbel model to describe the behaviour of extreme rainfall in SWA/Namibia. (Note, in the log-Gumbel model the variable x is replaced by the variable $z = \log x$ and the Gumbel model is then applied to the z values – the assumption is that z will be distributed according to Gumbel. Therefore x, *not* z (see comment by Mr. Adamson), is said to be distributed as log-Gumbel).

Selection of a probability distribution to describe the statistical behaviour of a large number of samples is seldom a straightforward task. To quote Mr. Adamson – "the choice must inevitably be made on empirical grounds." However, he does contradict himself to some extent by relying on a statistical test to substantiate his arguments concerning choice of distribution. In the case of the SWA/Namibia rainfall data no single frequency distribution could be expected to provide a satisfactory fit to the data from all 572 stations. Mr. Adamson's tests on the small sample of 23 stations illustrates this problem quite clearly.

The author, like Mr. Adamson, would never adopt a frequency distribution purely on the basis of chi-square tests. The log-Gumbel model was adopted by the author in a recent study of point rainfalls in South Africa (Midgley and Pitman, 1978). Therefore, it was considered desirable to adopt, if possible, the same model for the SWA/Namibia data so as to permit direct comparisons to be made. In addition to the chi-square tests, graphs were plotted for each record of maximum daily rainfall (MDR) to give a visual indication as to how well the log-Gumbel model fitted the data. In most cases the graphs correspond quite closely with what are often referred to as "eye-fits", i.e. the frequency distribution derived by ranking the data, assigning plotting positions and fitting a straight line to the data plotted on the adopted probability paper – in this case, log-Gumbel. This is perhaps what Mr. Adamson means by selecting a probabilistic model on empirical grounds.

No matter how good the fit or whatever the distribution adopted, there is always a great deal of uncertainty associated with extrapolation to frequencies b eyond the range of experience. In the SWA/Namibia lstudy the analyses indicated an increasing variance as one progresses towards the drier regions, as illustrated by Fig. 4 in the paper. However, this means that frequency curves for low mean annual precipitation (MAP) will exhibit steeper slopes, when plotted on extremal paper, than the slopes associated with areas of high MAP, with the result that the curves will "cross over" if extrapolated too far. Before plotting the depth-duration-frequency diagram (Fig. 5 in the paper), therefore, the MDR-frequency relationships covering the full range of MAPs were plotted on the same graph and the tails of the curves for the low MAPs adjusted downwards to ensure that no curves crossed. Whilst this graphical technique would not satisfy the purist, it does help to eliminate the possibility of generating the anomalous results that might be obtained by blind acceptance of purely numerical techniques.

**TABLE 1**
**COMPARISON OF 100 YEAR, 24 h POINT RAINFALLS IN THE SUMMER RAINFALL (INLAND) REGION**

| MAP (mm) | Point rainfall (mm) Wiederhold (using Gumbel) | Midgley & Pitman (using log-Gumbel) |
|---|---|---|
| 200 | 90 | 85 |
| 400 | 140 | 130 |
| 600 | 170 | 170 |
| 800 | 220 | 210 |
| 1 000 | 270 | 260 |

The author strongly disagrees with Mr. Adamson's implication that application of log-Gumbel invariably leads to considerable over-estimation of magnitudes associated with high return periods. As mentioned previously, it was the log-Gumbel model that was adopted by Midgley and Pitman (1978) for the purpose of updating the point rainfall-frequency diagram developed by Wiederhold (1969). Although Wiederhold employed the straightforward Gumbel model, the two studies exhibit markedly similar results, as shown in Table 1.

It would appear, therefore, that Mr. Adamson's claims of "dramatic" over-estimation are somewhat exaggerated. Although the choice of a probabalistic model is important, of perhaps greater importance is the technique of fitting such a model to one's data.

The author's comment that extreme rainfalls in SWA/Namibia, and possibly all arid areas in Southern Africa, could be higher than indicated by earlier work is based on the evidence of increased variance in the dry areas (Fig. 4). Previous studies by the Hydrological Research Unit (Van Wyk, 1965; Wiederhold, 1969), as well as studies based on the work of Hershfield (1972) and Bell (1969) relied on the assumption of constant variance throughout the region. In other words, the ratio of rainfall associated with say the 100-year event to that of say the 10-year event is deemed to be identical at all points.

The author makes no apology if, in endeavouring to allow for increased variance in the arid areas, he has tended to over, rather than under, estimate point rainfalls associated with long (e.g. 50 and 100 year) return periods. The results presented in the paper are intended for use by engineers responsible for the design and erection of structures to cope with flood flows. It is considered sound engineering practice to over, rather than under, design when faced with a situation of inadequate data. It must also be conceded that the network of hydrometeorological stations in SWA/Namibia, admirable though it is for such a sparsely populated country, is hardly adequate.

It is, after all, of purely academic interest to the unfortunate victims of catastrophic floods such as occurred in Laingsburg whether the causative storm was a once in a hundred, a thousand or even a million year event. While it may be intellectually stimulating to argue the pros and cons of different methodologies, one must never lose sight of the fact that real people are going to be affected by one's choice of design storm or flood.

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